

**Key Words:** Absorption-H, Mass Transfer-H, Reactions-H, Rates-H, Chlorine-I, Halogens-I, Liquid Phase-I, Hydrolysis-I, Chlorine-A, Water-A, Columns (Process)-J, Absorption-, Wetted Wall-, Pressure-F, Partial-, Flow-F, Rates-, Velocities-F, Rates-G, Reactions-G, Constants-G, Mass Transfer-G, Coefficients-G, Predictions-H, Mass Transfer-I, Rates-I, Coefficients-I, Unsteady State-J, Theory-J, Penetration-J, IBM-709-J, Computers-J.

**Abstract:** The rate of absorption of chlorine into water has been studied in a short wetted-wall column. This system is characterized by absorption with simultaneous liquid-phase chemical reaction. The effect of chlorine partial pressure and liquid flow rate on the ratio of the absorption coefficient accompanied by hydrolysis reaction to that for physical absorption alone is presented. The experimental results are compared with penetration theory predictions obtained by the use of an IBM-709 digital computer.

**Reference:** Brian, P. L. T., J. E. Vivian, and A. G. Habib, *A.I.Ch.E. Journal*, **8**, No. 2, p. 205 (May, 1962).

**Key Words:** Critical-H, Temperature-H, Pressure-H, Mixtures-H, Binary-I, Ternary-I, Properties (Characteristics)-I, Physical Properties-I, Thermodynamics-I, State (Conditions)-I, Phase Diagrams-I, Phases-I, Diagrams-I, Methane-A, Ethane-A, Butane-A, Hydrocarbons-A, Cells-J, Pressure-, Visual-.

**Abstract:** An experimental unit has been designed and constructed for the study of critical phenomena in multicomponent systems. The unit has been built to utilize a new visual PVT cell of the liquid piston type, capable of operation at temperatures up to 300°F. and pressures up to 10,000 lb./sq. in. Critical temperatures and pressures have been determined for two binary mixtures of ethane and n-butane, and for six different compositions of the methane-ethane-n-butane system. The results for the six ternary mixtures are compared with critical values predicted by methods available in the literature. The complete critical locus for the ternary system also is presented.

**Reference:** Forman, J. C., and George Thodos, *A.I.Ch.E. Journal*, **8**, No. 2, p. 209 (May, 1962).

**Key Words:** Vector Analysis-H, Analyzing-H, Operations-I, Stages-I, Mass Transfer-I, Mixing-I, Separation-I, Mathematics-I, Differences-J, Methods-J, Techniques-J.

**Abstract:** A fundamental unified approach from vectorial considerations is presented from which the usual rectangular and triangular difference point methods are derived as special cases. The approach is then extended to yield other difference point methods including some of higher order. The results are applicable to blending, distillation, extraction, foam fractionation, and other operations.

**Reference:** Lemlich, Robert, and R. A. Leonard, *A.I.Ch.E. Journal*, **8**, No. 2, p. 214 (May, 1962).

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## Reply

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The derivations presented by Vohr and by Lamb and White illustrate the use of the momentum and energy balances in specifying the irreversible losses, and as emphasized by Lamb and White the viscous dissipation losses are not to be confused with the term involving the total wall shearing force.

In our momentum balance we have chosen to relate a static pressure drop for the two-phase flow into components of head and momentum, and a residual which is called *frictional two-phase pressure drop*. For the past fifteen years the attention of most investigators working with two-phase flow has been with this residual term [as presented in our communication *A.I.Ch.E. Journal*, **7**, 174 (1961)].

What rewards would be gained by attempting to focus the attention on the dissipation term have yet to be spelled out.

## A Note on Unsteady Forced Convection Heat Transfer

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Recently Siegel (2) and Sparrow and Siegel (3) investigated unsteady heat transfer in laminar and turbulent flow where the disturbance was due to time dependent wall temperatures. The unsteady problem with time dependent entrance conditions caused by disturbances on the upstream side of the thermal entrance is perhaps of greater interest to chemical engineers, since the method of solution can be extended to solve transient problems involving chemical reactions in flow systems. This will be considered briefly here.

Since the solution for unsteady laminar heat transfer may be obtained in an identical manner, turbulent heat transfer in tubes will be discussed. The solutions satisfy the transient integral energy equation; however they con-

verge to exact steady state solutions to the differential energy equation at  $t = 0$  and  $t = \infty$ . For simplicity a step change in entrance temperature is assumed to occur at  $t = 0$ . Hence the initial condition is the steady state temperature distribution corresponding to given values of  $T_o$  and  $T_w$ . Since fluid properties are considered constant, the velocity distribution is unaffected by the change in entrance temperature.

The transient energy equation is

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\alpha + \epsilon_k) \frac{\partial T}{\partial r} \right] \quad (1)$$

for fully developed turbulent flow in tubes. In dimensionless form and integrated from 0 to  $r_o^+$ , Equation (1) becomes

$$\begin{aligned} \frac{1}{r_o^+} \frac{\partial}{\partial t} \left[ \int_0^{r_o^+} r^+ \theta dr^+ \right] \\ + \frac{1}{2r_o^+} \frac{\partial}{\partial x^+} \left[ \int_0^{r_o^+} r^+ u^+ \theta dr^+ \right] \\ = \frac{r_o^+}{N_{Pr}} \left( \frac{\partial \theta}{\partial r^+} \right)_{r_o^+} \end{aligned} \quad (2)$$

where

$$\theta = \frac{T - T_w}{T_o - T_w}$$

Initially the exact steady state solution obtains and is given by

$$\theta(0, x^+, r^+) = \sum_{n=1}^{\infty} C_n \phi_n(r^+) e^{-(\lambda_{Bn}^2 x^+ / N_{Re})} \quad (3)$$

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**Key Words:** Diffusion-I, Porosity-I, Solids-I, Properties (Characteristics)-I, Physical Properties-I, Rates-I, Mass Transfer-I, Transport-I, Voids-H, Correlations-H, Helium-A, Nitrogen-A, Bulk Density-F, Physical Properties-F, Properties (Characteristics)-F, Rates-G, Diffusion-G, Pellets-J, Particles-J, Catalysts-J, Sliver-J, Metals-J, Salts-J.

**Abstract:** Diffusion rates have been measured at atmospheric pressure and room temperature for helium and nitrogen in pelleted silver catalysts. Pellets were prepared from three silver salts giving different micropore characteristics; for each kind of material data were obtained for five pellet densities corresponding to macropore volume fractions from 0 up to 0.7. Labyrinth factors were evaluated and correlated with void fractions. A single-constant expression, derived from the generalized Maxwell equation, has been used to correlate the measured results over the entire range of macropore volume fractions.

**Reference:** Masamune, Shinobu, and J. M. Smith, *A.I.Ch.E. Journal*, **8**, No. 2, p. 217 (May, 1962).

**Key Words:** Thermal Conductivity-H, Properties (Characteristics)-H, Physical Properties-H, Transport-H, Rates-H, Heat-H, Heat Transfer-H, Fluids-I, Gases-I, Mixture-I, Binary-, Ternary-, Temperature-F, Thermal Conductivity-G, Cells-J, Thermal Conductivity-, Cylinders-J, Correlations-H, Empirical-, Equations-I, Predictions-I, Thermal Conductivity-I, Properties (Characteristics)-I, Physical Properties-I, Kinetic Theory-J.

**Abstract:** Correlations based upon empirical equations derived from kinetic theory have been developed for the thermal conductivity of gas mixtures; the results are compared with experimental data for 226 binary mixtures over temperatures from 0° to 774°C. Thermal conductivities of ten gases and selected binary and ternary mixtures were measured in a concentric silver cylinder cell over the temperature range from 100° to 540°C.

**Reference:** Cheung, Henry, L. A. Bromley, and C. R. Wilke, *A.I.Ch.E. Journal*, **8**, No. 2, p. 221 (May, 1962).

**Key Words:** Viscosity-H, Physical Properties-H, Properties (Characteristics)-H, Correlations-H, Gases-I, Fluids-I, Polarity-I, Pressure-I, Temperature-F, Molecular Weight-F, Critical-F, Constants-, Compressibility-F, Viscosity-G, Analysis-J, Dimensional-.

**Abstract:** The viscosity behavior at normal pressures (0.2 to 5 atm.) of 53 polar gases has been examined, and equations representing the dependence of viscosity on temperature for these gases have been developed. A dimensional analysis approach is employed to determine the relationship between viscosity and molecular weight, temperature, and the critical constants of the substance. A defined viscosity parameter is presented as a function of  $z_c$  for polar gases which do and do not exhibit hydrogen bonding.

**Reference:** Stiel, L. J., and George Thodos, *A.I.Ch.E. Journal*, **8**, No. 2, p. 229 (May, 1962).

**Key Words:** Coalescing-I, Agglomeration-I, Liquid Phase-I, Drops (Droplets)-H, Globules-H, Dispersions-H, Mixtures-H, Solutions (Mixtures)-H, Water-A, Toluene-E, Hydrocarbons-E, Intensity-F, Mixing-F, Frequency-G, Coalescing-G, Models-J.

**Abstract:** An experimental technique is described for estimating the coalescence frequency of droplets in an agitated liquid-liquid system. Results are presented for dilute aqueous dispersions in toluene. Quantitative estimates of the coalescence frequency are based upon a simple model proposed to describe the coalescence process. The coalescence frequency is shown to increase markedly with increase in impeller speed and to a lesser extent with increase in the dispersed-phase volume.

**Reference:** Madden, A. J., and Glenn L. Damerell, *A.I.Ch.E. Journal*, **8**, No. 2, p. 233 (May, 1962).

**Key Words:** Fluidizing-H, Profiles-I, Bulk Density-I, Density-I, Physical Properties-I, Properties (Characteristics)-I, Distribution-I, Correlations-I, Height-I, Velocities-F, Size-F, Height-F, Bulk Density-G, Pressure Drop-G, Beds-J, Fluidized-, Radiation-J, Gamma Rays-J.

**Abstract:** An  $\gamma$ -ray attenuation method has been used to study the axial fluidizing bulk density profile for an air-solid aggregative fluidized bed. Two distinct density zones, a constant zone and a falling zone, were observed except for the bed fluidizing at very high air velocity. The average densities of these zones have been correlated as functions of operating variables. The effects on fluidization characteristics of particle composition in beds containing a mixture of two sizes of particles also have been studied. Pressure drop is presented as a function of friction factor. Three different ways of defining bed height are discussed.

**Reference:** Fan, Liang-tseng, Chau Jen Lee, and Richard C. Bailie, *A.I.Ch.E. Journal*, **8**, No. 2, p. 239 (May, 1962).

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where  $C_n$ ,  $\phi_n(r^+)$ , and  $\beta_n$  have been calculated (3) up to  $n = 7$  for several combinations of  $N_{Re}$  and  $N_{Pr}$ . Since the wall temperature is constant, after the step change in entrance temperature

$$\theta(t^+, x^+, r_o^+) = 0; \frac{\partial \theta}{\partial r^+}(t^+, x^+, 0) = 0; \theta(t^+, 0, r^+) = \frac{T_1 - T_w}{T_o - T_w} \quad (4)$$

Now the solution to Equation (2) is taken as

$$\theta = \sum_{n=1}^{\infty} C_n \phi_n(r^+) e^{\frac{4\beta_n^2 x^+}{N_{Re}}} F_n(t^+, x^+) \quad (5)$$

Substituting Equation (5) into (2) one obtains

$$\begin{aligned} \frac{1}{r_o^{+2}} \left[ \int_0^{r_o^+} r^+ \phi_n(r^+) dr^+ \right] \frac{\partial F_n}{\partial t^+} \\ + \frac{1}{2r_o^+} \left[ \int_0^{r_o^+} r^+ u^+ \phi_n(r^+) dr^+ \right] \\ \left[ \frac{\partial F_n}{\partial x^+} - \frac{4\beta_n^2}{N_{Re}} F_n \right] \\ = \frac{r_o^+}{N_{Pr}} \left[ \frac{d\phi_n}{dr^+} \right]_{r_o^+} F_n \quad (6) \end{aligned}$$

From the Sturm-Liouville equation associated with the steady state problem it is easily shown that

$$\beta_n^2 = - \frac{r_o^+ N_{Re} \left( \frac{d\phi_n}{dr^+} \right)_{r_o^+}}{2 N_{Pr} \int_0^{r_o^+} r^+ u^+ \phi_n dr^+}$$

Hence Equation (6) is equivalent to (1)

$$\begin{aligned} \frac{dt^+}{\frac{1}{r_o^{+2}} \int_0^{r_o^+} r^+ \phi_n(r^+) dr^+} \\ = \frac{dx^+}{\frac{1}{2r_o^+} \int_0^{r_o^+} r^+ u^+ \phi_n(r) dr^+} = \frac{dF_n}{0} \quad (7) \end{aligned}$$

Therefore

$$\begin{aligned} F_n = 1, \quad t^+ \leq A_n x^+ \\ F_n = \frac{T_1 - T_w}{T_o - T_w} t^+ \geq A_n x^+ \quad (8) \end{aligned}$$

where

$$A_n = \frac{4\beta_n^2}{r_o^{+2} N_{Re}} \frac{N_{Pr} \int_0^{r_o^+} r^+ \phi_n(r^+) dr^+}{(d\theta_n/dr^+)_{r_o^+}} \quad (9)$$

Some values of  $A_n$  have been tabulated by Sparrow and Siegel (3).

From Equations (5) and (8) it is seen immediately that the initial condition and the boundary conditions are satisfied and further that the correct

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steady state solution for  $t = \infty$  is obtained.

Fortunately, as seen from the above analysis, once the steady state quantities have been determined they may be used to solve not only the time dependent wall temperature problem, but also that associated with a transient entrance temperature.

The author has found that the technique described above can be used in solving the transient diffusion equation with chemical reaction where the entrance concentration is time dependent. This problem will be treated in detail in a subsequent paper.

#### NOTATION

$A_n$  = coefficient defined in Equation (9)  
 $C_n$  = expansion coefficient  
 $d$  = tube diameter  
 $F_n$  = function of  $x$  and  $t$  defined by Equation (5)  
 $k$  = thermal conductivity  
 $N_{Pr}$  = Prandtl number,  $(\nu/\alpha)$   
 $N_{Re}$  = Reynolds number,  $(Ud/\nu)$   
 $r$  = radial coordinate;  $r_o$ , tube radius;  $r^*$ , dimensionless radial coordinate,  $\left(\frac{r\sqrt{\tau_o/\rho}}{\nu}\right)$ ;  $r_o^*$  dimensionless tube radius

$T$  = temperature;  $T_o$ , temperature of entering fluid at  $t = 0$ ;  $T_1$ , temperature of entering fluid at  $t > 0$ ;  $T_w$ , constant wall temperature  
 $t$  = time  
 $u$  = fluid velocity;  $U$ , bulk mean velocity;  $u^*$ , dimensionless fluid velocity,  $u/\sqrt{\tau_o/\rho}$   
 $x$  = axial coordinate;  $x^*$ , dimensionless axial coordinate,  $(x/d)$

#### Greek Letters

$\alpha$  = molecular thermal diffusivity,  $(k/\rho C_p)$   
 $\beta_n$  = eigenvalues  
 $\epsilon_h$  = eddy diffusivity of heat  
 $\theta$  = dimensionless temperature,  $(T - T_w)/(T_o - T_w)$   
 $\nu$  = kinematic viscosity  
 $\rho$  = density  
 $\tau_o$  = wall shear stress  
 $\phi_n$  = eigenfunction

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3. Sparrow, E. M., and R. Siegel, *J. Heat Trans.*, **82**, series C, No. 3, p. 170 (1960).